

Important!

1. Please submit all hand-written solutions to me by 1 June 2002 as I would be leaving for France for one month after that.
2. You can still continue to submit your solutions to me by email (asteea@singnet.com.sg) when I am in France (that's assuming I have Internet access in the town of Besancon)
3. Nope, I am not that rich, I cannot afford so many books, so what I will do instead is I will get some stuff from France and Europe for those complete solutions given to me.
4. If no complete solution is received, then I'll just give it to the one who's closest, to encourage all of you to write your proofs down (if it helps at all...)

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- 1 There are 99 space stations. Each pair of space stations is connected by a tunnel. There are 99 two-way tunnels, and all the other tunnels are strictly one-way tunnels. A group of 4 space stations is called **connected** if one can reach each station in the group from every other station in the group without using any tunnels other than the 6 tunnels which connect them. Determine the maximum number of connected groups.
 - 2 Let ABC be a triangle and let ω be a circle passing through A and C . Sides AB and BC meet ω again at D and E , respectively. Let γ be the incircle of the **circular** triangle EBD and let S be its center. Suppose that γ touches the arc DE at M . Prove that the angle bisector of $\angle AMC$ passes through the incenter of triangle ABC .
 - 3 Consider all numbers of the form $3n^2 + n + 1$, where n is a positive integer.
 - (a) How small can the sum of the digits (in base 10) of such a number be?
 - (b) Can such a number have the sum of its digits (in base 10) equal to 2002?
 - 4 Define the sequence $\{x_n\}_{n \geq 0}$ by $x_0 = 0$ and

$$x_n = \begin{cases} x_{n-1} + \frac{3^{r+1}-1}{2} & \text{if } n = 3^r(3k+1), \\ x_{n-1} - \frac{3^{r+1}+1}{2} & \text{if } n = 3^r(3k+2), \end{cases}$$

where k and r are nonnegative integers. Prove that every integer appears exactly once in the sequence.